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# Lecture 5: Basic Optics for Optical Sensing-III

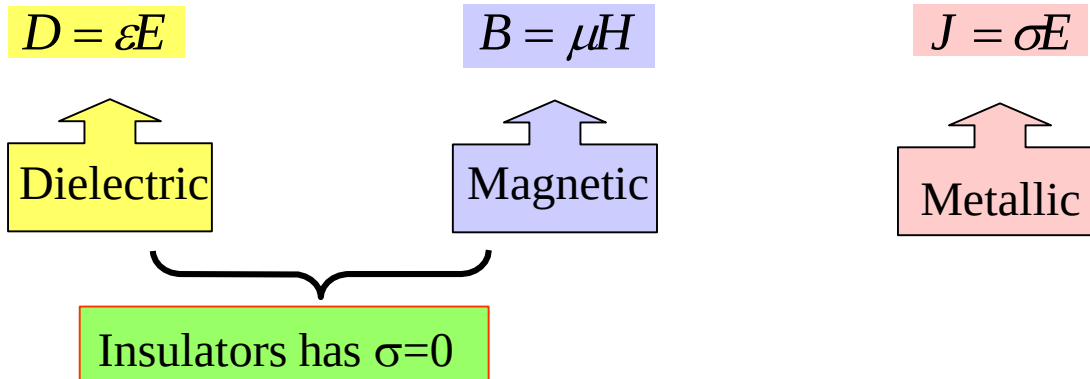
## Reflection and transmission at interface: Fresnel equations

Sachin Kumar Srivastava  
Department of Physics



# Electromagnetic Waves in Matter

- EM waves in free space is the most simple case
- Matter is characterized by dielectric, magnetic or conducting properties
- **Constitutive relations:**



# Ideal Insulators

- Insulators are easy

- Just replace  $\epsilon_0 \rightarrow \epsilon$   $\mu_0 \rightarrow \mu$

- Speed of light becomes

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \rightarrow \frac{c}{n} = \frac{1}{\sqrt{\epsilon \mu}}$$

NB:  $\epsilon$  and  $\mu$  may be  $\omega$ -dependent in dispersive medium!

$$\begin{aligned} \nabla \cdot \mathbf{E} &= 0 & \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} & \nabla \times \mathbf{B} &= \epsilon \mu \frac{\partial \mathbf{E}}{\partial t} \end{aligned}$$

$$n = \sqrt{\epsilon_r \mu_r}$$

Index of refraction

- Plane wave solutions looks the same

$$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$$

$$\mathbf{B} = \mathbf{B}_0 e^{i(\mathbf{k} \cdot \mathbf{x} - \omega t)}$$

$$\omega = \frac{1}{\sqrt{\epsilon \mu}} k$$

- Power density is

$$\langle |S| \rangle = \frac{E_0^2}{2Z}$$

$$Z \equiv \sqrt{\frac{\mu}{\epsilon}}$$

Impedance of the material

E/H

$$\begin{aligned} \nabla^2 \mathbf{E} - \frac{1}{v^2} \ddot{\mathbf{E}} &= 0 \\ \nabla^2 \mathbf{B} - \frac{1}{v^2} \ddot{\mathbf{B}} &= 0 \end{aligned}$$

# Boundary Conditions

At the interface between two media the following boundary conditions apply for the EM field:

1. Tangential components of the electric field are continuous.

$$E_{1tan} = E_{2tan}$$

2. Tangential components of the magnetic field are discontinuous.

$$H_{2tan} - H_{1tan} = 4\pi J_s / c$$

3. The normal components of the magnetic induction B are always continuous.

$$B_{1norm} = B_{2norm}$$

4. The normal components of the electric displacement D are continuous in the absence of charge and they change abruptly by an amount proportional to the surface charge density:

$$D_{2norm} - D_{1norm} = 4\pi\rho_s$$

# Interface Between Two Insulators

- EM waves crosses boundary between two insulators

- e.g., light (in air) hits a glass window

- At the boundary,

- $\mathbf{E}$  must be continuous  $\mathbf{E}_I + \mathbf{E}_R = \mathbf{E}_T$
  - $\mathbf{H}$  must be continuous  $\mathbf{H}_I + \mathbf{H}_R = \mathbf{H}_T$

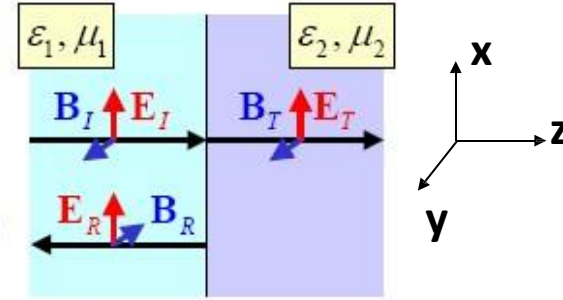
- Use the relationship  $H = \frac{E}{Z}$

- Continuity of  $\mathbf{H}$  becomes  $\frac{E_I}{Z_1} - \frac{E_R}{Z_1} = \frac{E_T}{Z_2}$

Does this look familiar?

Note sign!

Use:  $\vec{k} \times \vec{E} = \omega \vec{B} = ck\mu\vec{H}$  and equate x, y and z components to find out that  $E_y$  and  $H_x$  are in opposite signs when propagating in the +z direction while  $E_x$  and  $H_y$  have the same sign and the opposite for the reflected wave!



# Fresnel Reflection and Transmission

- Problem reduced to  $E_I + E_R = E_T$  and  $\frac{E_I}{Z_1} - \frac{E_R}{Z_1} = \frac{E_T}{Z_2}$ 
  - Similar to LC transmission lines
- Solution:  $E_R = \frac{Z_2 - Z_1}{Z_1 + Z_2} E_I$   $H_R = \frac{Z_1 - Z_2}{Z_1 + Z_2} H_I$   $S_R = -\left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2 S_I$   
 $E_T = \frac{2Z_2}{Z_1 + Z_2} E_I$   $H_T = \frac{2Z_1}{Z_1 + Z_2} H_I$   $S_T = \frac{4Z_1 Z_2}{(Z_1 + Z_2)^2} S_I$ 
  - Determined by the impedance
- Reflectivity  $R$  = how much power is reflected

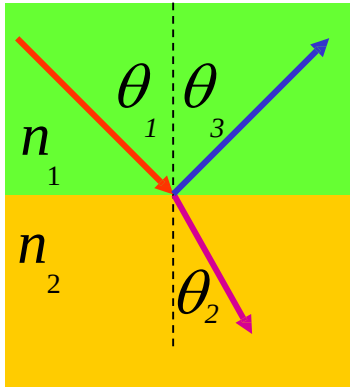
$$R = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2$$

Example:  $Z_{\text{air}} \approx Z_0 = 377\Omega$   $Z_{\text{glass}} \approx 250\Omega$

➡  $R_{\text{air} \rightarrow \text{glass}} = 0.04$  About 4% reflection

Write it down in terms of the refractive indices !

# Oblique Incidence



In a similar manner to the normal incidence case and using Snell's law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

we get:

$$r_s = \frac{n_1 \cos \theta_1 - n_2 \cos \theta_2}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$$

$$r_p = \frac{n_2 \cos \theta_1 - n_1 \cos \theta_2}{n_2 \cos \theta_1 + n_1 \cos \theta_2}$$

$$R_{p,s} = |r_{p,s}|^2$$

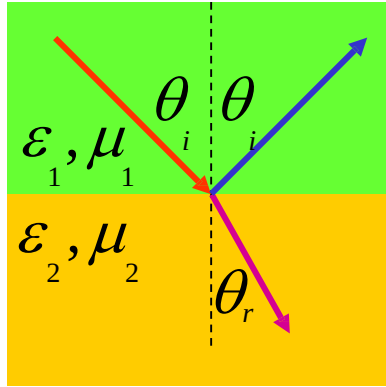
$$t_s = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2}$$

$$t_p = \frac{2n_1 \cos \theta_1}{n_1 \cos \theta_2 + n_2 \cos \theta_1}$$

$$T_{p,s} = \frac{n_2 \cos \theta_2}{n_1 \cos \theta_1} |t_{p,s}|^2$$

**Fresnel Coefficients**

# Oblique Incidence – (with magnetic permeability)



Use:

$$Z_{1,2} = \sqrt{\frac{\mu_{1,2}}{\epsilon_{1,2}}}$$

$$r_s = \frac{Z_2 \cos \theta_i - Z_1 \cos \theta_r}{Z_2 \cos \theta_i + Z_1 \cos \theta_r}$$

$$r_p = \frac{Z_2 \cos \theta_r - Z_1 \cos \theta_i}{Z_2 \cos \theta_r + Z_1 \cos \theta_i}$$

$$t_s = \frac{2Z_2 \cos \theta_i}{Z_2 \cos \theta_i + Z_1 \cos \theta_r}$$

$$t_p = \frac{2Z_2 \cos \theta_i}{Z_2 \cos \theta_r + Z_1 \cos \theta_i}$$

# Summary

1. The boundary conditions at the interface of two media for EM wave propagation were discussed.
2. The propagation of an EM wave at the interface of two media was studied rigorously and Fresnel coefficients were derived.

**Thank You**